

# Short or long-term contract? Firm's optimal choice

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**Abstract** This article studies the behaviour of a firm searching to fill a vacancy. The main assumption is that the firm can offer two different kinds of contracts to the workers, either a short-term contract or a long-term one. The short-term contract acts as a probationary stage in which the firm can learn about the worker. After this stage, the firm can propose a long-term contract to the worker or it can decide to look for another worker. We show that, if the short-term wage is fixed endogenously, it can be optimal for firms to start a working relationship with a short-term contract, but that this policy decreases unemployment and welfare. On the contrary, if the wage is fixed exogenously, this policy could be optimal also from a welfare point of view.

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## 1 Introduction

The share of temporary work in total employment has been increasing in Europe in recent years. For instance, in the OECD, temporary employment as a proportion of dependent employment has grown from 11.2% in 1998 to 12.3% in 2008. This evolution has not been homogeneous across countries. Thus, since 1998 the share of temporary workers has increased in some nations such as Germany and France and decreased in others like the UK and Spain.<sup>1</sup>

Short-term contracts are often considered a measure of labour market flexibility. They make the returns to entrepreneurial activities less dependent of institutional rigidities such as employment restrictive legislation and trade union activity. In addition, short-term contracts can be viewed as a screening device. In this perspective, job matches can be interpreted as an experience good, in the tradition of Jovanovic (1979, 1984). Accordingly, firms would use a short-term contract as a probationary period, allowing them to select the right worker for the job.

In spite of this hypothesis, there is not a clear consensus at the empirical level about the impact of temporary contracts on employment and welfare, notwithstanding the fact that most of these papers apply a similar econometric framework: a discrete-time proportional hazard model that relates the transition from temporary to permanent employment to a number of individual and job-specific characteristics; see, among others, Güel and Petrongolo (2007) and Dolado et al. (2002) for Spain, Gagliarducci (2005) for Italy, Salvanes (1997) for seven OECD countries, Booth et al. (2002) for Britain and D'Addio and Rosholm (2005) for the European Community. Thus, Booth et al. (2002) and Salvanes (1997) find evidence that flexible labour markets facilitate the transition to permanent work while the evidence is mixed for D'Addio and Rosholm (2005) who find that very short contracts provide higher chances of labour market exclusion, Güel and Petrongolo (2007) who estimate that conversion rates from transitory to permanent employment are below 10% and Dolado et al. (2002) who observe that temporary works in Spain are highly persistent during the 90s despite several reforms attempting to reduce it. Gagliarducci (2005) finds that the duration of temporary contracts increases the probability of obtaining a permanent job but, in turns, this probability is a decreasing function of the number of interruptions in the short-term contracts.<sup>2</sup>

The aim of this paper is to propose a theoretical model in order to understand the factors influencing the impact of temporary contracts on employment and welfare by analyzing the behavior of a firm searching to fill a vacancy under a short-term contract and a long-term one. More specifically, we construct a model with heterogeneous workers and homogeneous firms. Each firm has two possibilities: either it offers only long-term contracts (LTCs) or it offers a short-term contract (STC) to begin with, eventually switching to a long-term one if it is satisfied with the productivity of the job-workers pairs. We assume that firms offer a long-term

<sup>1</sup> See OECD (2009) for a more detailed information of the evolution of temporary contracts in OECD countries.

<sup>2</sup> Additional to this, a recent paper by Ederveen and Thissen (2007) for the new EU member states finds that the impact of the rigidity of labor market institutions on unemployment is mixing.

wage for the job and that workers either take it or leave it. In this model, we assume that the short-term wage is fixed exogenously, whatever the worker type. The implication of this assumption is analyzed in the last section, where we evaluate the case where the short-term wage is set by the firms.<sup>3</sup>

We show that, when short-term wage is exogenous and sufficiently high, STCs can be optimal for both firms and social welfare. This is because a high short-term wage pushes every worker to accept a STC. Through these contracts, firms will be able to screen workers thereby hiring only the most qualified on a long-term basis. From a social welfare viewpoint, the screening has a negative impact on employment. However, such adverse effect, can be compensated by a higher productive efficiency.

When firms are posting the short-term wages (and we allow firms to use contingent wages), in equilibrium, STC may be optimal for the firms. STCs are profitable only if the costs arising from the probability of being unmatched are compensated by the surplus from the long-term matching. The higher the workers' unemployment benefits, the lower the surplus that can be obtained by waiting to find a good worker. However, in this case STCs are never optimal from a welfare viewpoint.

Moreover, we show that the regime with fixed short-term wage dominates, in terms of welfare, the regime with posted short-term wage, provided that the short-term wage exceeds a threshold value.

We also establish that, when the short-term wage is exogenously given, the STC wage is higher than the LTC one, which is equal to the worker's reservation wage given the income received when unemployed. This is because unemployment benefits are the outside option for workers faced with a STC. Similarly, the exogenous short-term wage is the outside option of a worker faced with a LTC.

In determining the optimal firms' behaviour, an important role is played by the short-term wage. We will mostly focus on the more interesting and relevant case when this salary is exogenously given. The last section of the paper will summarize the result for the case when firms also set the salary.

Our model shares some similarities with Wasmer (1999) and Cahuc and Postel-Vinay (2002),<sup>4</sup> who have introduced temporary jobs in matching models based on the classic equilibrium models of the labour market, built on Diamond (1982), Mortensen (1982), and Pissarides (2000).

Unlike this article, they concentrate on the macroeconomic effect of STCs. Wasmer (1999) shows that, in a model with exogenous job destruction, firms are more willing to use STCs, in periods of low growth, and that this increases employment. Cahuc and Postel-Vinay (2002) show that, in a model with endogenous job destruction, the combination of temporary jobs and firing restrictions may be both inefficient in terms of aggregate welfare and inadequate

<sup>3</sup> The implications of exogenous wages on economic performance have already been analysed by Zagler (2005). In his model, endogenous salaries obtained from the bargaining process between individual firms and unions could generate a distorted remuneration system that pays too much to the innovation sector and too little to the existing stock of knowledge. Zagler (2005) suggests that optimal salaries may be obtained from centralized wage pacts and not by government policy.

<sup>4</sup> See also Paolini (2007).

as a weapon to fight unemployment. Their results follow from the fact that the share of temporary jobs transformed into permanent jobs is decreasing in the level of firing costs.

The model is formally introduced in Sect. 2. Sections 3, 4 and 5, study the optimal contract choice, present the main results and analyze the welfare effects with exogenous short-term wages. Section 6 analyzes the main results and the welfare effects when these wages are endogenous. Section 4 concludes.

## 2 The model

### 2.1 Workers and firms

Workers are characterized by a real-value parameter defining the worker's type,  $x$ , distributed according to a continuous distribution function  $F(x)$  with full support on  $[0,1]$ . Its density denoted by  $f(x)$ .

Time is discrete and  $t = 0, 1, \dots + \infty$ .  $\beta$  is the probability that a worker leaves the market ("dies") when going from period  $t$  to period  $(t + 1)$ . We assume that, in each period, new workers enter the market. Their types are distributed so that the actual distribution of live workers is time-invariant, i.e., a worker of type  $x$  enters the market if and only if a worker of type  $x$  dies. This is a strong assumption, but it allows to keep the analysis sufficiently simple. The main results of the paper are independent of it.

In each period firms and workers meet. Firms offer a job-contract pair and workers can either accept or reject the offer. If the offer is accepted, production takes place. At the end of the period, wages are paid and output is sold.

As in Albrecht and Axel (1984), we assume that workers can earn different levels of income when unemployed,<sup>5</sup>  $b(x)$ . However, while Albrecht and Axel (1984) assume that homogenous workers can have just two possible levels of unemployment incomes, here we assume that there is a continuum of unemployment income levels, depending on workers' types. More precisely, we assume that unemployment income is proportional to the worker's type, i.e., that  $b(x) = \gamma x$ , with  $\gamma \in [0,1]$ .

We assume that the number of homogenous firms is higher than the number of workers,  $M > N$ . At each moment in time, a firm can have either a filled position or a vacancy. An active firm with a filled position employs one worker and obtains revenue by selling its output.

If the position is filled, an exogenous layoff arrives at each period with probability  $\beta$  (this is because in each period  $t$ , a matched worker dies with probability  $\beta$ ). Moreover, we assume that, if a firm leaves the market, another firm enters the market.

For simplicity, for each firm, the production function is:

$$Y = f(x) = xy \tag{1}$$

<sup>5</sup> It can be interpreted as including the value of leisure and home production, net of search costs. This wide notion of unemployment income also justifies the assumption that benefits are related to the type of worker.

where  $x$  is the type of the worker and  $y$  the technology of the homogenous firms. To simplify notation, we set  $y = 1$ .

If there is a vacancy, in each  $t$ , firms can create a position without cost. At each meeting in the search market, firms are not able to observe the type of the worker (hence, their unemployment income). We allow firms to offer a probationary contract (lasting one period) to the workers. During this period, firms learn their workers' type and decide whether or not to offer them a long-term contract.

All the agents (firms and workers) have a common discount factor, denoted  $0 \leq \delta \leq 1$ . Impatience also captures search costs.

## 2.2 Search and matching

Total employment is  $N_a = N(1 - u_a)$ , where  $N$  is the labour force and  $u_a$  the unemployment rate. These values depend on  $a$ , the firm's choice of the contract.<sup>6</sup>

Unemployed workers are matched with the recruiting firms according to a simple random matching technology,  $\alpha$ , that is assumed to be independent of the number of participants in the search market.

The rate at which firms find an unemployed worker, denoted  $q$ , will depend on  $\alpha$ , the matching technology, and on  $u_a$  and  $v_a$  (unemployment and vacancy rate). We simply assume that  $q$  is a fixed proportion of the unemployment/vacancy ratio.<sup>7</sup> Hence,

$$q = \alpha \frac{Nu}{Mv}.$$

## 2.3 Optimal firm behaviour

In determining the optimal firm behaviour, an important role is played by the short-term wage  $w_O$ . We will mostly focus on the more interesting and relevant case when  $w_O$  is exogenously given, for instance because it is fixed by law.

The last section of the paper will summarize the result for the case when firms also post  $w_O$ . Here we treat the short term wage  $w_O$  as exogenous.

For a firm, the policy,  $a$ , is the choice of the contract to offer to the workers. We assume that it is impossible for a firm to fire a worker before the expiration of the contract (or that to fire a worker is infinitely costly).

The firm has two policy options:

(L) It supplies a long-term employment contract at a wage rate  $w_L(x)$ , contingent on the type ( $a = L$ ).

(SL) It supplies a short-term (one-period) contract to begin with, switching, possibly, in the following period, to a long-term employment contract at a wage rate  $w_{SL}(x)$ , contingent on the type ( $a = SL$ ).

<sup>6</sup> We give the definition of contract,  $a$ , in the next section.

<sup>7</sup> Given that  $M > N$ , evidently  $v > u$ .

The advantage of the latter option is that the firm will have full information about the worker's type when it offers him/her the long-term contract. Note also that the permanent contract under the first policy is equivalent to a temporary one with an option to remain in employment; see for example Booth et al. (2000). Thus, a permanent contract is contingent on wages and the key difference between this and a temporary one is that workers cannot be fired.

Contracts are chosen by the firm so as to maximize its profit subject to the participation of workers, corresponding to the expected utility of a worker not accepting the contract.

Let

$$V_a(x) = b(x) + \delta(1 - \beta)[\alpha W_a(x) + (1 - \alpha)V_a(x)] \quad (2)$$

be the expected utility of a type- $x$  unemployed where  $a \in \{L, SL\}$ ,  $x \in [0, 1]$  and  $(1 - \beta)$  is the probability that the worker will survive to the next period.

An ( $L$ ) policy is fully characterized by a wage rate  $w$ , meaning that a worker of type  $x$  will receive a wage  $w_L(x)$ . The expected utility for a worker is implicitly defined by

$$W_L(x) = w_L(x) + \delta(1 - \beta)W_L(x). \quad (3)$$

Given our simple matching technology (and the assumption  $M > N$ ), where  $\alpha$  is the probability that a workers receives a job offer, a worker accepts it if and only if  $W_L(x) \geq V_a(x)$ . Let  $\rho_L$  be the (measurable) subset of workers accepting the contract.

An ( $SL$ ) contract is fully characterized by a short-term wage  $w_0$ , a long-term wage rate  $w_{SL}(x)$  and a subset  $\sigma$  of workers for which the firm is willing to extend the contractual relation over the long-period. The expected utility for a worker is implicitly defined by

$$W_{SL}(x) = \begin{cases} w_0 + \delta(1 - \beta)V_{SL}(x), & \text{if } x \notin \sigma, \\ w_0 + \delta(1 - \beta)W_L(x; w) & \text{if } x \in \sigma. \end{cases} \quad (4)$$

Clearly, a worker accepts if and only if  $W_{SL}(x) \geq V_{SL}(x)$ . Let  $\rho_{SL}$  be the (measurable) subset of workers accepting the contract.

### 2.3.1 The firm policy choice problem

Knowing  $w_0, w_{SL}(x), w_L(x), \sigma$  and  $w_0$  we can define the problem firms' policy choice.

Let  $J(x)$  be the expected profit of firm with a long-term contract, that is,

$$J(x) = (x - w_a(x)) + \delta(1 - \beta)J(x)$$

where  $a \in \{L, SL\}$  and  $w_a(x)$  is the long-term wage that, by hypothesis, is posted by firms and that workers either take or leave.

Moreover, let  $\Pi_L$  denote the expected discounted payoff of a firm searching to fill a vacancy with a policy  $L$ , with probability  $(1 - q)$ , that it does not succeed and it

will try again to match in the next period. If the firm decides to offer directly a LTC ( $a = L$ ), it will hire, by hypothesis, every worker.

$$\Pi_L = q \int_{x \in \rho_L} J(x)f(x)dx + \delta \left[ 1 - q \int_{x \in \rho_L} f(x)dx \right] \Pi_L(w_0) \quad (5)$$

where  $\rho_L$  is the (measurable) subset of workers accepting the contract.

And, let  $\Pi_{SL}$  denote the expected discounted payoff of a firm searching to fill a vacancy. Given the short-term wage  $w_0$ , the expected profit of the firm is

$$\begin{aligned} \max_{\sigma} \Pi_{SL} = & q \int_{x \in \rho_{SL}} (x - w_0)f(x)dx + q\delta(1 - \beta) \int_{x \in \sigma_{SL} \cap \rho_{SL}} J(x)f(x)dx \\ & + \delta \left[ 1 - q \int_{x \in \rho_{SL}} f(x)dx + q\delta(1 - \beta) \int_{x \notin \sigma_{SL} \cap \rho_{SL}} f(x)dx \right] \Pi_{SL} \end{aligned} \quad (6)$$

(notice that, given the continuity of the distribution function, single points have zero probability of occurrence).

The first term is the one-period expected profit from a short-term job, as the contract will be accepted by workers of the subset  $\rho_{SL}$ ; the second term is the expected profit from an extension to a long-term contract, which occurs only for types in the subset  $\sigma_{SL}$ ; the third term collects all cases in which the firm supplies again a short-term contract, either because the the matching was unsuccessful, or because the short-term contract was rejected, or because the firm refuses to extend the contract to permanent position.

If the firm chooses to start with a STC,  $a = SL$ , during the probationary stage, it will learn the worker's type. In this stage, the worker will receive a short-term wage:  $w_0$ . In period 2, the firm will propose a LTC only to the workers in the subset  $\sigma_{SL}$ . Given that the firm learns the worker's type, future wages will be contingent on the worker's type:  $w_a = w_{SL}(x)$ .

The firm maximizes its profits by choosing policy  $SL$  if  $\Pi_{SL} \geq \Pi_L$ , otherwise policy  $L$  will be optimal.

### 3 Optimal contract choice

We study the optimal contract choice in four steps:

- we determine the subset of workers ( $\rho_L, \rho_{SL}$ ) accepting the contract proposed. To this end we need to determine the wages;
- we determine the subset of workers ( $\sigma_{SL}$ ) accepted by firm after a short-term contract;
- we determine the unemployment rates;
- we find the optimal contract choice.

### 3.1 Wage posting

The profit maximizing firm chooses the wage subject to  $W_a(x) \geq V_a(x)$ , with  $a \in \{L, SL\}$ . Since the firm has no incentive to offer workers anything over and above the minimum required to make him/her to accept its offer (see Diamond (1971)), this condition reduces to

$$W_a = V_a. \quad (7)$$

Let's first consider an (L) policy. In this simple wage setting set-up, the condition (7), where  $W_L$  is given by (3) and  $V_L$  is given by (2), implies that  $w_L$  is driven to the worker's reservation wage:

$$w_L(x) = b(x) = \gamma x. \quad (8)$$

Notice that  $w_L$  is type-contingent. This implies that the subset of workers accepting the contract,  $\rho_L$ , is the full support  $[0,1]$ . Also, observe that, even with type-contingent wages, firms would rather hire high productivity (and highly paid) workers because, per period, their profits are  $(1 - \gamma)x$ .

Consider an (SL) policy. Condition (7), where  $W_{SL}$  is given by (4) and  $V_L$  is given by (2), immediately implies that the long-term wage associated with SL is<sup>8</sup>

$$w_{SL}(x) = \frac{\gamma x + \alpha \delta (1 - \beta) w_O}{1 + \alpha \delta (1 - \beta)} \quad (9)$$

**Proposition 1** *Under the maintained assumptions, given  $w_O$ , all the workers with  $x \in (x^+, 1]$  always reject the STC. For all the workers with  $x \in [0, x^+)$ ,  $w_O \geq w_{SL}(x) \geq b(x)$ , where  $x^+ = \frac{w_O}{\gamma}$ .*

*Proof* To establish the first proposition, observe that workers with  $x \in \sigma_{SL}$  will accept a SL if and only if  $W_{SL}(x) > V_{SL}(x)$  where  $W_{SL}(x)$  and  $V_{SL}$  are implicitly given by (4) and (2). By (9), the previous inequality is satisfied if and only if  $w_O > b(x) = \gamma x$ . Hence, all the workers with  $x \in (x^+, 1]$ , with  $x^+ = \frac{w_O}{\gamma}$ , will always reject the short-term contract. Given that, for all the workers accepting the SL,  $w_O > b(x)$ , the second claim follows immediately from (9).  $\square$

This result is of some interest. Due to the assumption  $b(x) = \gamma x$ , when  $w_O$  is exogenous, high productivity workers will reject short-term contracts, unless  $w_O$  is sufficiently high. However, high levels of  $w_O$  also imply high levels of  $w_{SL}(x)$ . This trade-off may make STC less profitable than LTC. The result is coherent with the wage formation theories suggesting that temporary workers are paid more than workers in long-term contract to compensate them for the less advantageous characteristics of temporary jobs. Moreover, in our model, the long-term wage obtained after a short-term contract (i.e., after the firm observes the type of the worker) is not driven to the worker's reservation wage. This is because, for the workers, the STC is an outside option: if the long-term wage is equal to the reservation wage, the worker will reject the firm's proposal and wait for the next STC proposal.

<sup>8</sup> Notice that this is only a hypothetical wage, as the firm will extend the contract only to workers in  $\sigma$ .



### 3.2 Search equilibrium in the STC case

Knowing that only workers with  $x \in [0, x^+)$  will always accept the short-term contract, the firm's strategy in the  $SL$  policy is to engage only workers in the set  $\sigma_{SL}$  after the probationary stage. Hence, for the stationary strategy profile  $(\sigma_{SL})$ , we define the expected payoff of a firm as  $\Pi_{SL}$ . We focus on equilibria in undominated strategies, where a firm accepts a worker of type  $x$  if and only if  $J^{SL}(x) > \Pi_{SL}$ .

**Proposition 2** *The optimal  $SL(x^-, x^+)$  is given by  $\sigma_{SL} = (x^-, x^+]$ , where  $x^- = \frac{q\delta^2(1-\beta)\left(\int_{x^-}^{x^+} xf(x)dx - \int_{x^-}^1 f(x)dx - \frac{D}{C}\right) + \frac{(1-\delta(1-\beta))}{C}\left[q\delta\left(\int_0^{x^+} xf(x)dx - w_0\right)\right]}{[1-\delta+qE\delta-q\delta^2(1-\beta)]}$ , while  $x^+ = \frac{w_0}{\gamma}$ ,  $E$  is the probability to meet a worker in the set  $[0, x^+)$ ,  $C = \frac{1-\gamma+q\delta(1-\beta)}{1+q\delta(1-\beta)}$  and  $D = \frac{q\delta(1-\beta)w_0}{1+q\delta(1-\beta)}$ .*

*Proof* See Appendix 1.  $\square$

Proposition 2 characterizes the search equilibrium in the case of STCs. In this equilibrium, firms partition workers who have accepted the short-term contract, into two subintervals, hiring permanently forever only workers whose ability is above the threshold  $x^-$ .<sup>9</sup>

An increase in  $w_0$  has a double effect on the economy. First, given that  $x^+ = \frac{w_0}{\gamma}$ , it increases the proportion of workers accepting a STC (and this has a positive effect on  $E$ ). Second, it pushes up the long-term wage  $w_{SL}$ , as we can see from (9).

To better understand this result, it is instructive to consider a simple example, where abilities are uniformly distributed on  $[0, 1]$ ,  $N = M$ ,  $q = \alpha = 0.8$ ,  $\gamma = 0.5$ ,  $\delta = 0.8$ , and  $\beta = 0.05$ . The next figure shows how the interval of workers accepting and accepted in the long-term job (i.e., with  $x \in [(x^+) - (x^-)]$ ) varies with  $w_0$ .

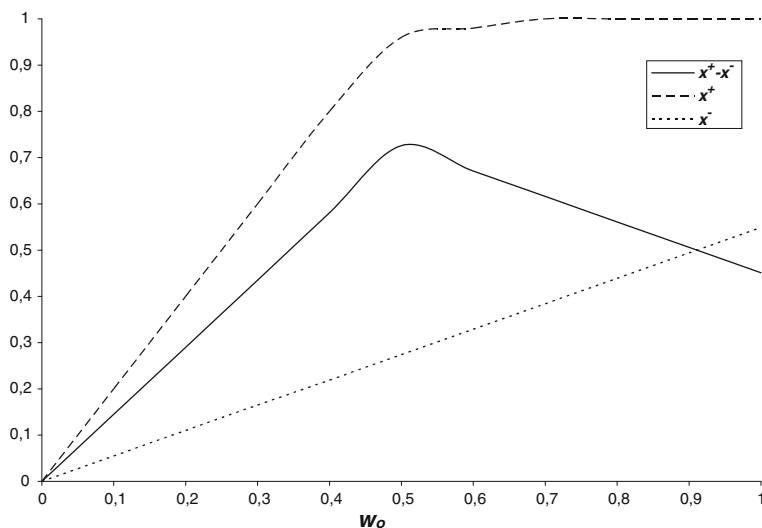
If  $w_0$  is low, the first one of the two effects discussed above dominates the second: only some workers accept a STC ( $x^+$  is low). On the other hand, firms are not very selective in screening the workers ( $x^-$  is low). When  $w_0$  is higher, the opposite is true:  $x^+$  takes a value near 1 (i.e., every worker accepts the STC), and  $x^-$  is also high (because the long-term wage is so high that firms only find it profitable to engage highly qualified workers). (Fig. 1)

### 3.3 Unemployment

In order to determine the unemployment rate, under the  $SL$  policy, we shall consider unemployment rates for workers with  $x \leq x^-$  (similarly for  $u_{x^- < x < x^+}^t$  and  $u_{x \geq x^+}^t$ ). Also, let's index with  $p_{SL}^t$  the proportion of workers engaged in a  $STC$ .

Given the policy  $SL$ , the expected value of  $u_{x^- < x < x^+}^t$ , evolves over time according to:

<sup>9</sup> Remember, however, that all the workers with  $x \in (\frac{w_0}{\gamma}, 1]$  will always reject the  $STC$ .



**Fig. 1** The interval of workers accepting/accepted in the long-term job

$$u_{x^- < x < x^+}^{t+1} = u_{x^- < x < x^+}^t (1 - \beta)(1 - \alpha) + \beta$$

$$p_{x^- < x < x^+}^t = u_{x^- < x < x^+}^{t-1} (1 - \beta)\alpha$$

because a fraction  $\alpha$  of the unemployed who survive (with a probability  $(1 - \beta)$ ) at  $t$  is expected to get a job, a fraction  $\beta$  of them is expected to die and to be replaced by a fraction  $\beta$  of the work force with  $x \in [x^-, x^+]$  (these individuals are necessarily unemployed in the first period of their life, given the time structure of the model).

Hence, the unique stationary state occurs at

$$\bar{u}_{x^- < x < x^+} = \frac{\beta}{\alpha(1 - \beta) + \beta} \quad (10)$$

and

$$\bar{p}_{x^- < x < x^+} = \bar{u}_{x^- < x < x^+} (1 - \beta)\alpha. \quad (11)$$

Similarly, for workers with  $x \leq x^-$ , the dynamics are described by.

$$u_{x \leq x^-}^{t+1} = u_{x \leq x^-}^t (1 - \beta)(1 - \alpha) + p_{x \leq x^-}^t (1 - \beta)(1 - \alpha) + \beta$$

and

$$p_{x \leq x^-}^t = u_{x \leq x^-}^{t-1} (1 - \beta)\alpha$$

The difference with respect to the previous case reflects the share of people employed at  $t$  in a STC that, given that  $x \leq x^-$ , become unemployed (if still alive) at  $(t + 1)$ .

Hence, the unique stationary state is described by

$$\bar{u}_{x \leq x^-} = \frac{\beta}{\alpha^2(1 - \beta) + \beta} \quad (12)$$

$$\bar{p}_{x \leq x^-} = \bar{u}_{x \leq x^-}(1 - \beta)\alpha \quad (13)$$

Obviously, the unemployment rate of the workers with  $x \geq x^+$ , is always 1, because they never accept employment.

Total unemployment is:

$$\bar{u}_{SL} = F(x^+ - x^-)\bar{u}_{x^- < x < x^+} + F(x^-)\bar{u}_{x \leq x^-} + F(1 - x^+) \quad (14)$$

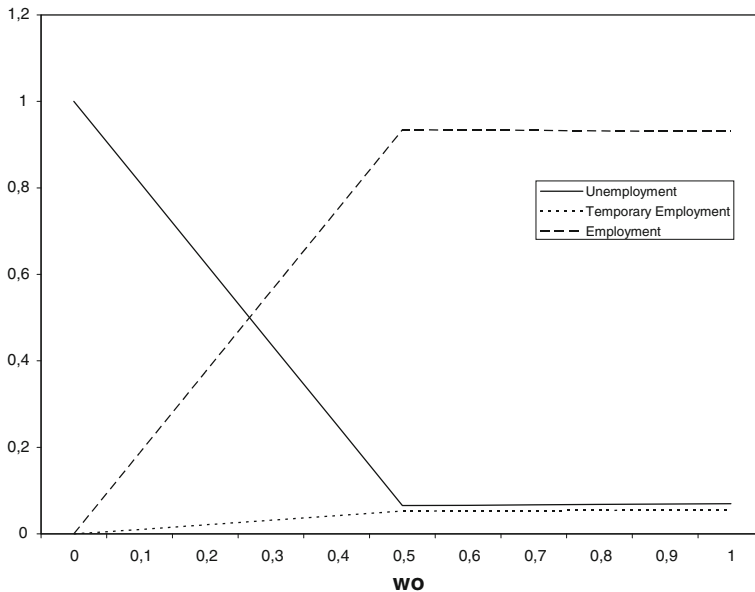
with a percentage  $\bar{p} = F(x^+ - x^-)\bar{p}_{x^- < x < x^+} + F(x^-)\bar{p}_{x \leq x^-}$  of workers are engaged in a one period contract.

Substituting (10) and (12) in (14), we find the value of the total unemployment:

$$\bar{u}_{SL} = 1 - \frac{\alpha(1 - \beta)}{\alpha(1 - \beta) + \beta} \left[ F(x^+) - \frac{\beta(1 - \alpha)}{\alpha^2(1 - \beta) + \beta} F(x^-) \right] \quad (15)$$

where the endogenous values  $x^+$  and  $x^-$  are given respectively by the Propositions 1 and 2.

Looking again at the numerical example above (see Fig. 2), the unemployment rate is decreasing in  $w_O$  if  $w_O < \gamma$ . After this point it starts increasing. When  $w_O = \gamma$ ,  $x^+ = 1$ , and so all the agents accept the temporary employment proposition. After this point, the increase of the short and, consequently, of the long-term wages pushes the firms to hire permanently only the most productive agent to compensate for the high cost of labour.



**Fig. 2** Unemployment, employment and temporary employment

We now turn to the analysis of the unemployment rate with the strategy  $L$ . Given the strategy  $L$ ,  $u_L^t$  is expected to evolve according to:

$$u_L^{t+1} = u_L^t(1 - \beta)(1 - \alpha) + \beta.$$

Hence, a stationary state occurs if and only if

$$\bar{u}_L = \frac{\beta}{(1 - \beta)\alpha + \beta} \quad (16)$$

It easy to check that  $\bar{u}_L \leq \bar{u}_{SL}$ . This is because, in policy  $SL$ , firms screen workers and engage long-term contracts only workers in the interval  $(x^-, x^+)$ . On the contrary, if  $a = L$ , this interval is the full support  $[0, 1]$ .

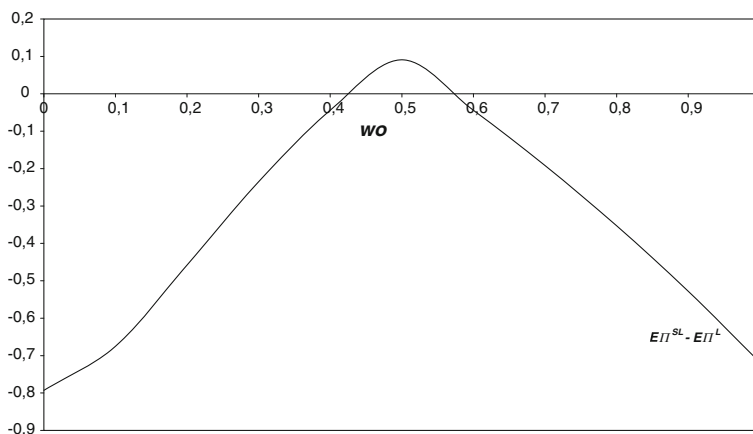
#### 4 Choice of the contract

Studying now the optimal contract choice: Policy  $SL$  is an optimal contract if  $\Pi_{SL} > \Pi_L$ , where  $\Pi^{SL}$  is defined in Appendix 1 by the Eq. 21, and,  $\Pi^L$  may be rewritten, from the Eq. 5 above, as:

$$\Pi_L = \frac{q\delta(1 - \beta)(1 - \gamma) \int_0^1 xf(x)dx}{[1 - (1 - \beta)\delta][1 - (1 - q)\delta]}$$

Looking again at the numerical example (see Fig. 3),  $a = SL$  is the firms' optimal contract if and only if  $w_O$  takes a value near  $\gamma$ .

For  $w_O$  less than  $\gamma$ , the more qualified workers will refuse to work. For  $w_O$  near  $\gamma$ , all qualified workers accept a short-term contract. This value of  $w_O$  implies a high level of  $w_{SL}$  but the high wages are compensated because firms can screen the workers in the first period. For  $w_O$  near 1, the screening cannot compensate the high wages and STC is suboptimal.



**Fig. 3** Firms' optimal contract choice

## 5 Welfare

We define welfare in terms of the present discounted value of output. The social planner is not interested in wages, since wages determine only the distribution of output and, by assumption, distributional considerations are excluded from the social welfare function. In our model, the critical issue is the firms' policy choice. We need to check if and when a *SL* policy can be optimal for both the firms and the social planner. We define as  $\Omega_a$  the welfare value with  $a = SL, L$ .

The value function of the planner, with  $a = L$ , satisfies the equation

$$\Omega_L = \sum_{t=0}^{+\infty} \delta^t N \left[ (1 - \bar{u}_L) \int_0^1 x f(x) dx + \bar{u}_L(\gamma) \int_0^1 x f(x) dx \right] \quad (17)$$

where  $\bar{u}_L$  is given by (16).

With  $a = SL$ , it satisfies the equation

$$\begin{aligned} \Omega_{SL} = & \sum_{t=0}^{+\infty} \delta^t N \left[ F(x^-)(1 - \bar{u}_{x^-}) \int_0^{x^-} x f(x) dx + (F(x^+) - F(x^-))(1 - \bar{u}_{x^- < x < x^+}) \right. \\ & \times \int_{x^-}^{x^+} x f(x) dx + F(x^-)(\bar{u}_{x^-}) \gamma \int_0^{x^-} x f(x) dx + (F(x^+) - F(x^-))(\bar{u}_{x^- < x < x^+}) \gamma \\ & \left. \times \int_{x^-}^{x^+} x f(x) dx + (1 - F(x^+))(\gamma) \int_{x^+}^1 x f(x) dx \right] \quad (18) \end{aligned}$$

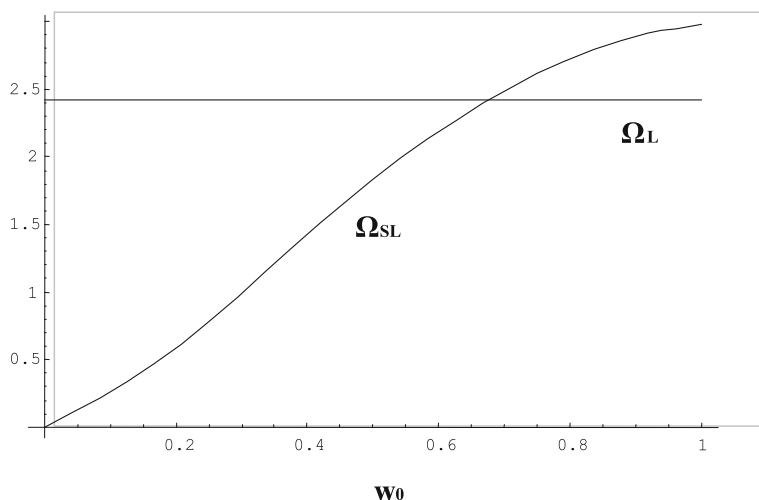
where  $\bar{u}_{x^- < x < x^+}$  is given by (10), while  $\bar{u}_{x \leq z}$  is given by (12). In each period, only workers with type  $x^- < x < x^+$  will be hired on LTC. On the contrary, workers with  $x \leq x^-$  will be hired exclusively on STC.

We need to check when

$$\Omega_{SL} \geq \Omega_L \quad (19)$$

Obviously the result depends upon the distribution  $F(\cdot)$  and the values of the parameters.

In our numerical example (see Fig. 4), an *SL* policy is optimal when the short-term wage is fixed at a value sufficiently high, so that  $x^+ = 1$ , i.e., so that all the workers enter the market for temporary jobs. In this case, the higher unemployment rate resulting from the firms' screening is compensated by the higher productive efficiency. Workers in the subset  $[0, x^-)$  will be always rejected by firms after the short-term contract. These workers will face a higher unemployment rate, nevertheless in the short-term relation they will obtain a wage  $w_O$  larger than their productivity.



**Fig. 4** Welfare comparison

## 6 Posted short-term wage

Let's now assume that the firm can also post the wage of the STC. The associated long-term wage is given by Eq. 8. When firms meet a worker for the first time, they don't know his type. We assume that the firm can choose to offer a (short-term) two-part wage. This wage would be  $w_O = [\tilde{w}(x) + (w(x) - \tilde{w}(x))]$ , where  $\tilde{w}(x)$  is the fixed component, while  $w(x)$  is contingent upon the type of the worker (which the firm will discover only at the end of the period). Hence, the firm will pay a salary  $w(\tilde{x})$  independent of the worker type, and, at the end of the period (after learning the type), it will pay a "bonus" equal to the difference between the worker's type and  $x$ , i.e.,  $[w(x) - \tilde{w}(x)]$ . From the proof of Lemma 1, it follows that a worker will accept a STC only if the expected utility of accepting this proposal is larger than the one of rejecting it. Therefore, if  $(w_O + \delta W_x^{SL}) > V_x^{SL}$ .

Clearly, at the equilibrium, given that types are distributed on  $[0,1]$ , the equilibrium value of  $\tilde{w}(x)$  will be 0 and  $w(x) = \gamma x$ . Hence, wages on long-term contracts (signed after the trial stage) and short-term wage are both driven to the worker's reservation wage:

$$w_{SL} = w_O = b(x) \quad (20)$$

Notice that, with a two-part wage, all the workers (also the high productivity ones with  $x \in (\frac{w_O}{\gamma}, 1]$ ) will enter the short-term market.

The main result for the case of endogenous short-term wage is summarized in the following proposition:

**Proposition 3** *When  $w_O$  is endogenous, firms only hire workers in the interval  $(z, 1]$ , where  $z = \frac{q\delta \left[ \delta(1-\beta) \left( \int_z^1 (x-z)f(x)dx \right) + (1-\delta(1-\beta)) \left( \int_0^1 xf(x)dx \right) \right]}{1-\delta+q\delta-q\delta^2(1-\beta)}$*

*Proof* See Appendix 2.  $\square$

If  $w_O$  is endogenous, as we have seen  $w_{SL} = w_O$  (see Eq. 20 above). In this case, every worker accepts a short-term contract offer. Moreover, firms will select only the agents in the subset  $(z, 1]$  for the long term work relation.

Looking again at the numerical example above, we find that  $\bar{u}_{sl} = 29\%$ ,  $\bar{p} = 4,7\%$  and  $z = 0.46$ . Compared to the case of endogenous  $w_O$ , the rate of unemployment is lower only when  $w_O$  is close to  $\gamma$ . Moreover, we find that  $a = SL$  is always optimal.

For the case of uniform distribution on  $[0,1]$ , it is straightforward to check that total welfare always attains a maximum with policy  $L$ . With a  $SL$  policy, firms screen workers. The screening process has a negative effect on unemployment and this generates a higher search cost. This effect could be compensated from the viewpoint of the workers on short-term contracts, because  $w_O > w_L$ . However, by definition, this is irrelevant from a social welfare viewpoint.

## 7 Conclusions

This paper shows that the introduction of a market for temporary labour contracts affects negatively unemployment and positively the productivity efficiency of firms. The positive impact compensates the negative one only if short-term wages are fixed exogenously at a sufficiently high value.

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## Appendix 1: Search equilibrium on the SL policy with $w_O$ exogenous

**Proof of Proposition 2:** First, observe that the set  $X$  of workers that the firm accepts in  $SL$  contract is always an interval  $(x^+, 1]$ , some  $x^+$ . Indeed, by the definition of undominated equilibrium,  $\frac{x}{1-\delta(1-\beta)} > \delta\Pi^{SL}(x)$ . Hence for any  $x' > x$ ,  $\frac{x'}{1-\delta(1-\beta)} \geq \delta\Pi^{SL}$ .

When  $w_O$  is fixed exogenously, by Proposition 1, all the workers with  $x \in (\frac{w_O}{\gamma}, 1]$  will always reject the STC.

Thus, the search problem faced by the firm may be rewritten, from (6) above, as:

$$\max \Pi^{SL} = \frac{\kappa[q\delta(1-\beta)(\int_0^{x^-} xf(x)dx - w_0)] + q\delta^2(1-\beta)^2[(\int_{x^-}^{x^+} xf(x)dx) - Ew_{sl}]}{\kappa[1-\delta(1-qE) - q\delta^2(1-\beta)(1-\int_{x^+}^1 f(x)dx)]}$$

or

$$\max \Pi^{SL} = \frac{\kappa [q\delta(1-\beta) \left( \int_0^{x^-} xf(x)dx - w_0 \right)] + q\delta^2(1-\beta)^2 \left[ C \left( \int_{x^-}^{x^+} xf(x)dx \right) - D \right]}{\kappa \left[ 1 - \delta + qE\delta - q\delta^2(1-\beta) \left( 1 - \int_{x^+}^1 f(x)dx \right) \right]} \quad (21)$$

where  $\kappa = (1 - \delta(1 - \beta))$ ,  $Ew_{sl} = \frac{\gamma \left( \int_{x^-}^{x^+} xf(x)dx \right) + \alpha\delta(1-\beta)w_0}{1 + \alpha\delta(1-\beta)}$ ,  $x^+ = \frac{w_0}{\gamma}$ ,  $E$  is the probability to meet a worker in the set  $[0, x^+]$ .

$$C = \frac{1 - \gamma + \alpha\delta(1 - \beta)}{1 + \alpha\delta(1 - \beta)} \text{ and } D = \frac{\alpha\delta(1 - \beta)w_0}{1 + \alpha\delta(1 - \beta)}.$$

The first-order conditions with respect to  $x^-$  are given by

$$\begin{aligned} & -x^-f(x^-)Cq\delta^2(1-\beta)^2(1-\delta(1-\beta)) \\ & \times \left[ \left( 1 - \delta + qE\delta - q\delta^2(1-\beta) \left( 1 - \int_{x^-}^1 f(x)dx \right) \right) \right] \\ & + q\delta^2(1-\beta)f(x)(1-\delta(1-\beta)) \left[ q\delta^2(1-\beta)^2 \left[ C \left( \int_{x^-}^{x^+} xf(x)dx \right) - D \right] \right. \\ & \left. + (1-\delta(1-\beta)) \left[ q\delta(1-\beta) \left( \int_0^{x^+} xf(x)dx - w_0 \right) \right] \right] = 0 \end{aligned}$$

or

$$x^- = \frac{q\delta^2(1-\beta) \left( \int_{x^-}^{x^+} xf(x)dx - \int_{x^-}^1 f(x)dx - \frac{D}{C} \right) + \frac{\kappa}{C} \left[ q\delta \left( \int_0^{x^+} xf(x)dx - w_0 \right) \right]}{[1 - \delta + qE\delta - q\delta^2(1-\beta)]} \quad (22)$$

To check that the solution is unique, observe that the left-hand side of Eq. 22 is increasing in  $x^-$ , with range  $(0,1)$ . On the other hand, the right-hand side of Eq. 22 is decreasing in  $x^-$ , falling from

$$\frac{q\delta^2(1-\beta) \left( \int_{x^-}^{x^+} xf(x)dx - \int_{x^-}^1 f(x)dx - \frac{D}{C} \right) + \frac{\kappa}{C} \left[ q\delta \left( \int_0^{x^+} xf(x)dx - w_0 \right) \right]}{[1 - \delta + qE\delta - q\delta^2(1-\beta)]}$$

to  $\frac{q\delta^2(1-\beta) \left( -\frac{D}{C} \right) + \frac{\kappa}{C} \left[ q\delta \left( \int_0^{x^+} xf(x)dx - w_0 \right) \right]}{[1 - \delta + qE\delta - q\delta^2(1-\beta)]}$ . Hence a unique solution to (22) exists.

## Appendix 2: Search equilibrium on the SL policy with $w_0$ endogenous

**Proof of Proposition 3:** As in the Proof of Proposition 1, the search problem faced by the firm may be rewritten as



$$\max \Pi^{SL} = \frac{\kappa \left[ q\delta(1-\beta)(1-\gamma) \left( \int_0^1 xf(x)dx \right) \right] + q\delta^2(1-\beta)^2 \left[ (1-\gamma) \left( \int_z^1 xf(x)dx \right) \right]}{\kappa \left( 1 - \delta(1-q) - q\delta^2(1-\beta)(1 - \int_z^1 f(x)dx) \right)}$$

where  $\kappa = (1 - \delta(1 - \beta))$  with  $w_{SL} = w_O = \gamma x$ .

The first-order conditions with respect to  $z$  are given by

$$\begin{aligned} & -zf(z)q\delta^2(1-\beta)^2(1-\gamma) \\ & \times \left[ (1-\delta(1-\beta)) \left( 1 - \delta + q\delta - q\delta^2(1-\beta)(1 - \int_z^1 f(x)dx) \right) \right] \\ & + q\delta^2(1-\beta)f(z)(1-\delta(1-\beta)) \left[ q\delta^2(1-\beta)^2(1-\gamma) \left( \int_z^1 xf(x)dx \right) \right] \\ & + (1-\delta(1-\beta)) \left( q\delta(1-\beta)(1-\gamma) \left( \int_0^1 xf(x)dx \right) \right) \right] = 0 \end{aligned}$$

or

$$z = \frac{q\delta}{1-\delta+q\delta-q\delta^2(1-\beta)} \left[ \delta(1-\beta) \left( \int_z^1 (x-z)f(x)dx \right) + \kappa \left( \int_0^1 xf(x)dx \right) \right] \quad (23)$$

To check that the solution is unique, observe that the left-hand side of Eq. 23 is increasing in  $z$ , with range  $(0,1)$ . On the other hand, the right-hand side of Eq. 23 is decreasing in  $z$ , falling from

$$\begin{aligned} & \frac{q\delta}{1-\delta+q\delta-q\delta^2(1-\beta)} \left[ \delta(1-\beta) \left( \int_0^1 (x-z)f(x)dx \right) + \kappa \left( \int_0^1 xf(x)dx \right) \right] \\ & \text{to } \frac{q\delta}{1-\delta+q\delta-q\delta^2(1-\beta)} \left[ \kappa \left( \int_0^1 xf(x)dx \right) \right]. \end{aligned}$$

Hence a unique solution to (23) exists.

## References

- Albrecht J, Axel B (1984) An equilibrium model of search unemployment. *J Polit Econ* 92:824–40
- Booth AL, Francesconi M, Frank J (2000) Temporary jobs: who gets them, what are they worth, and do they lead anywhere? ILR working papers 054, Institute for Labour Research
- Booth AL, Francesconi M, Frank J (2002) Temporary jobs: stepping stones or dead ends? *Econ J* 112(480):189–213
- Cahuc P, Postel-Vinay F (2002) Temporary jobs, employment protection and labour market performance. *Labour Econ* 9:63–91
- D’Addio A Cr, Rosholm M (2005) Exits from temporary jobs in Europe: a competing risks analysis. *Labour Econ* 12:449–468

- Diamond P (1982) Wage determination and efficiency in search equilibrium. *Rev Econ Stud* 49(2):217–27
- Dolado JJ, García-Serrano C, Jimeno JF (2002) Drawing lessons from the boom of temporary jobs in Spain. *Econ J* 112:270–295
- Ederveen S, Thissen L (2007) Can labour market institutions explain high unemployment rates in the new EU member states? *Empirica* 34:299–317
- Gagliarducci S (2005) The dynamics of repeated temporary jobs. *Labour Econ* 12(4):429–448
- Güell M, Petrongolo B (2007) How binding are legal limits? Transitions from temporary to permanent work in Spain. *Labour Econ* 14:153–183
- Jovanovic B (1979) Job matching and the theory of turnover. *J Polit Econ* 87(5):972–990
- Jovanovic B (1984) Matching, turnover, and unemployment. *J Polit Econ* 92(1):108–122
- Mortensen DT (1982) The matching process as a non-cooperative/bargaining game. In: McCall JJ (eds) *The economics of information and uncertainty*. University of Chicago Press, Chicago
- OECD (2009) *Employment Outlook*. Paris
- Paolini D (2007) Search and the firm's choice of the optimal labor contract. CRENoS 2007/08
- Pissarides CA (2000) *Equilibrium unemployment theory*. MIT, Cambridge
- Salvanes KG (1997) Market rigidities and labour market flexibility: an international comparison. *Scand J Econ* 99:315–333
- Wasmer E (1999) Competition for jobs in a growing economy and the emergence of dualism in employment. *Econ J* 109:349–371
- Zagler M (2005) Wage pacts and economic growth. *J Econ Stud* 32(5):420–434